Nuclear-Structure Theory for $\beta\beta$ Decay

Challenges and Opportunities

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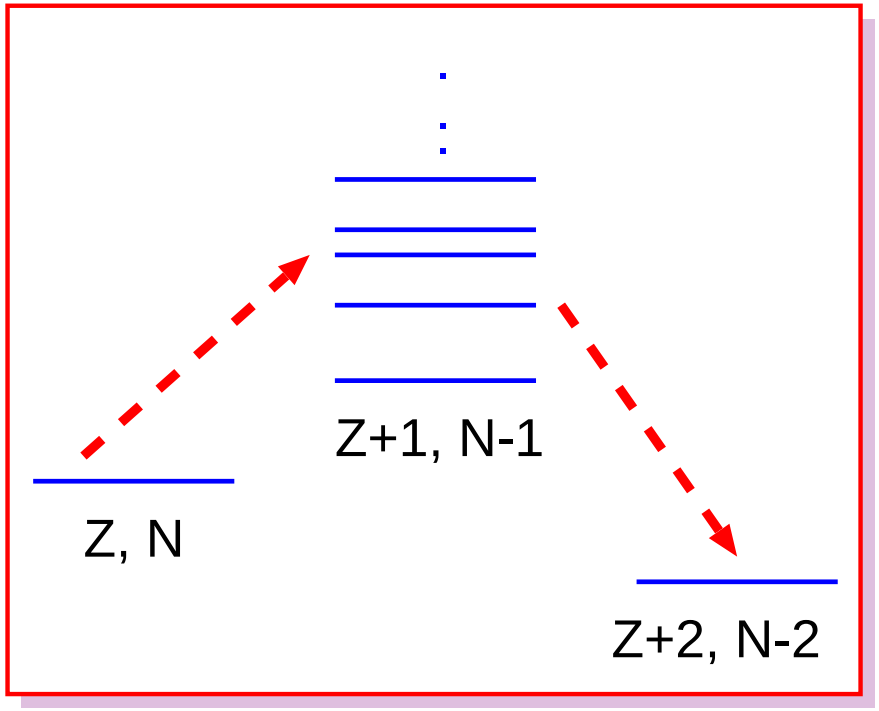
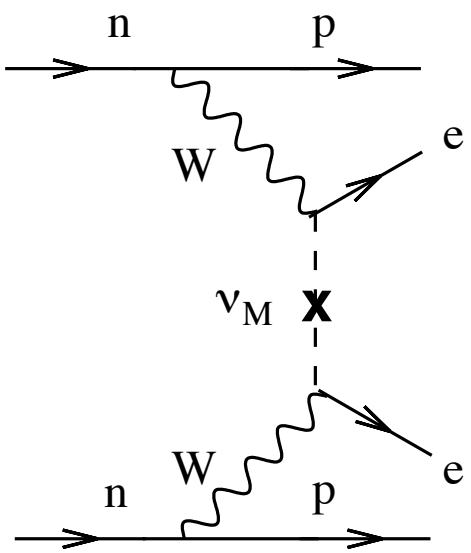
Neutrinolesss Double-Beta Decay

If energetics are right (ordinary beta decay forbidden)...



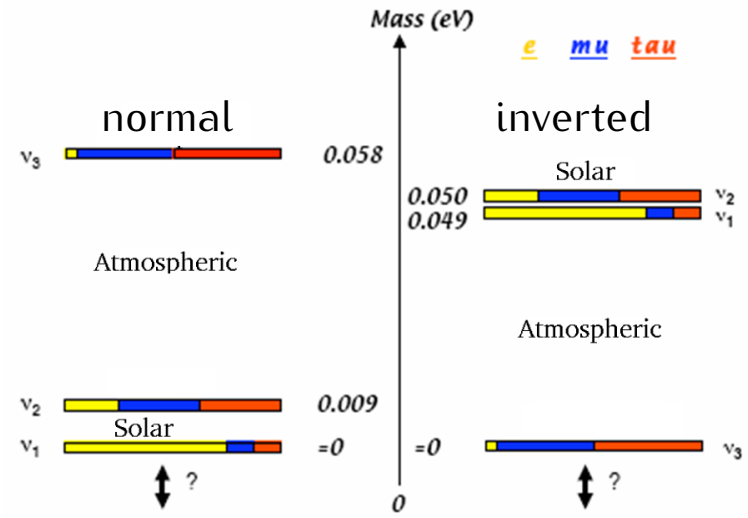
and neutrinos are their own antiparticles...

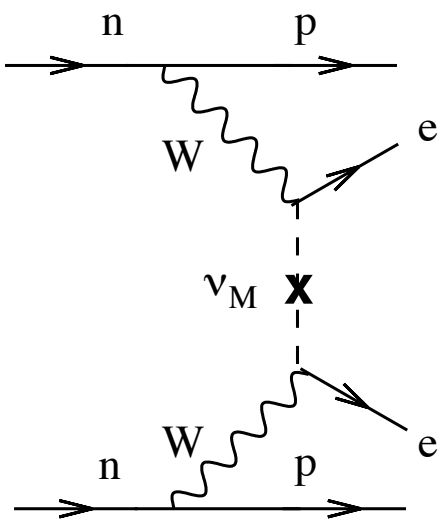


can observe two neutrons turning into protons, emitting two electrons and nothing else.

Why Double-Beta Decay Is Important

besides the ability to determine whether $\nu = \bar{\nu}$



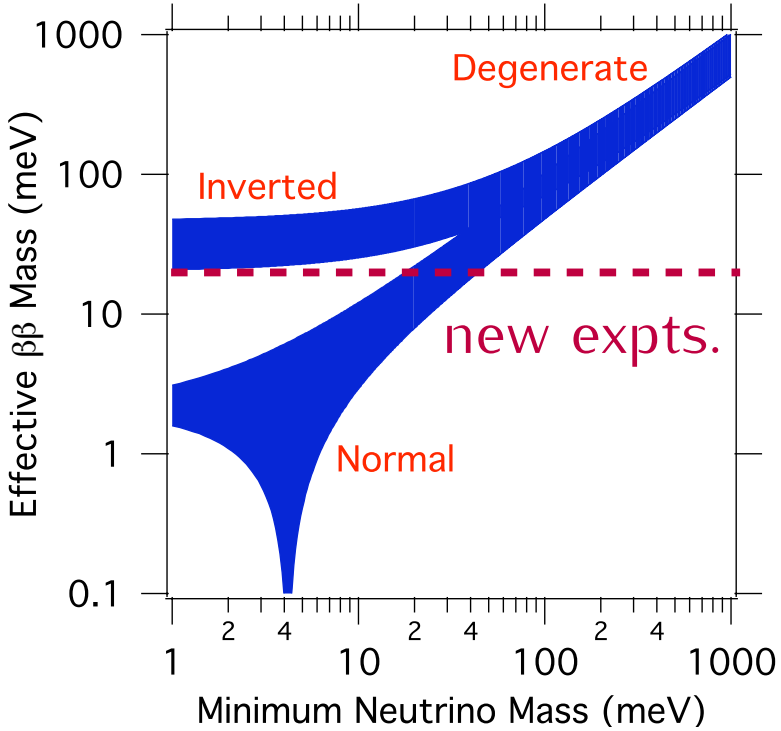


In usual scenario, rate depends on square of "effective neutrino mass"

$$m_{\text{eff}} \equiv \sum_i m_i U_{ei}^2$$

If lightest neutrino is light:

- $m_{\text{eff}} \propto \sqrt{\Delta m^2_{\text{sol}}}$ normal
- $m_{\text{eff}} \propto \sqrt{\Delta m^2_{\text{atm}}}$ inverted



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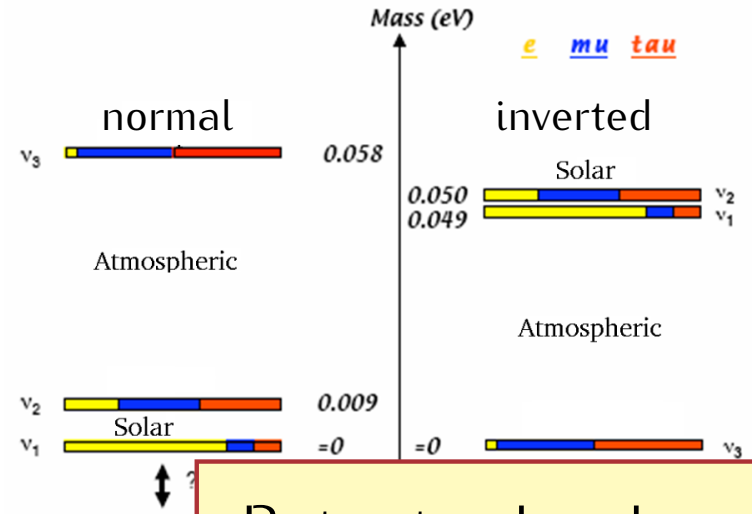

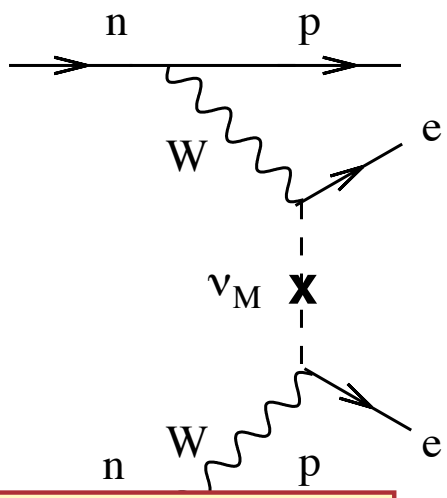

But rate also depends on a nuclear matrix element.

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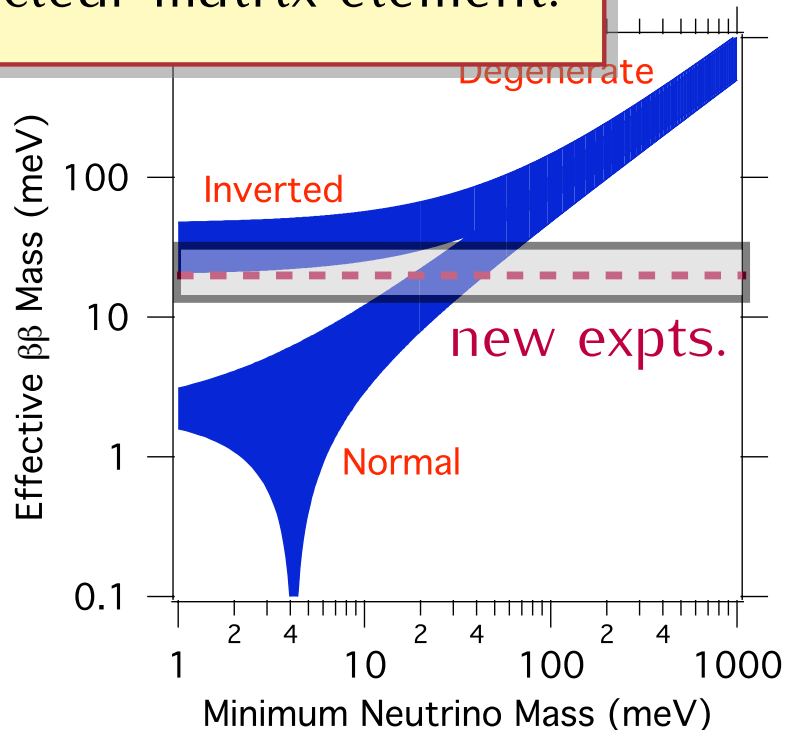
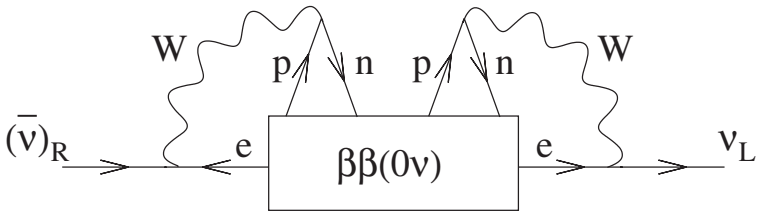
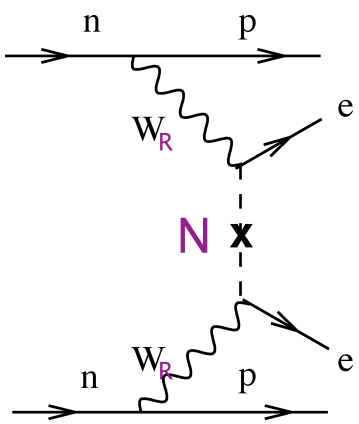


Other Mechanisms Can Contribute

If neutrinoless decay occurs then ν's are Majorana, no matter what:



but light neutrinos may not drive the decay:



Exchange of heavy right-handed neutrino in left-right symmetric model.

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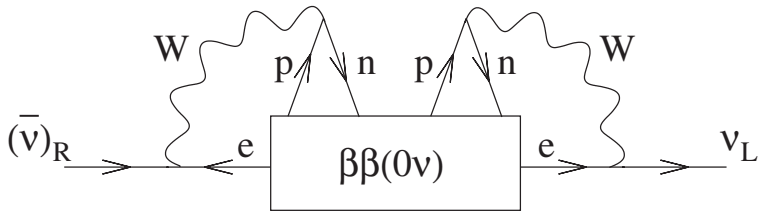

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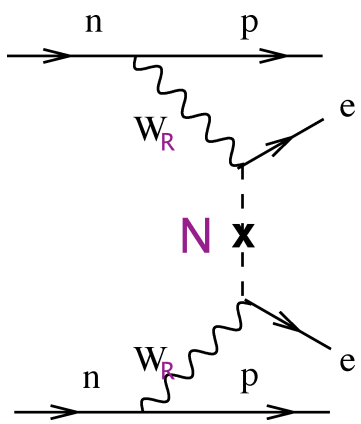

Exchange of heavy right-handed neutrino in left-right symmetric model.

Amplitude of exotic mechanism:

$$\frac{Z_{0\nu}^{\text{light}}}{Z_{0\nu}^{\text{heavy}}} \approx \left(\frac{W_L}{W_R}\right)^4 \left(\frac{m_{\text{eff}}}{\langle q^2 \rangle}\right) m_N \qquad \langle q^2 \rangle \approx 10^4 \ \text{MeV}^2$$

$$\approx \mathbf{1} \quad \text{if} \quad m_R \approx 1 \text{ TeV} \quad \text{and} \quad m_{\text{eff}} \approx \sqrt{\Delta m^2_{\text{atm}}}$$

So exotic processes can occur with roughly the same rate as light-ν exchange. Untangling will require multiple measurements and accurate nuclear matrix elements for all processes.

Light-ν-Exchange Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \dots$$

with

$$M_{0\nu}^{GT} = \langle F | \sum_{i,j} H(r_{ij}, \overline{E}) \vec{\sigma}_i \cdot \vec{\sigma}_j \tau_i^+ \tau_j^+ | I \rangle + \dots$$

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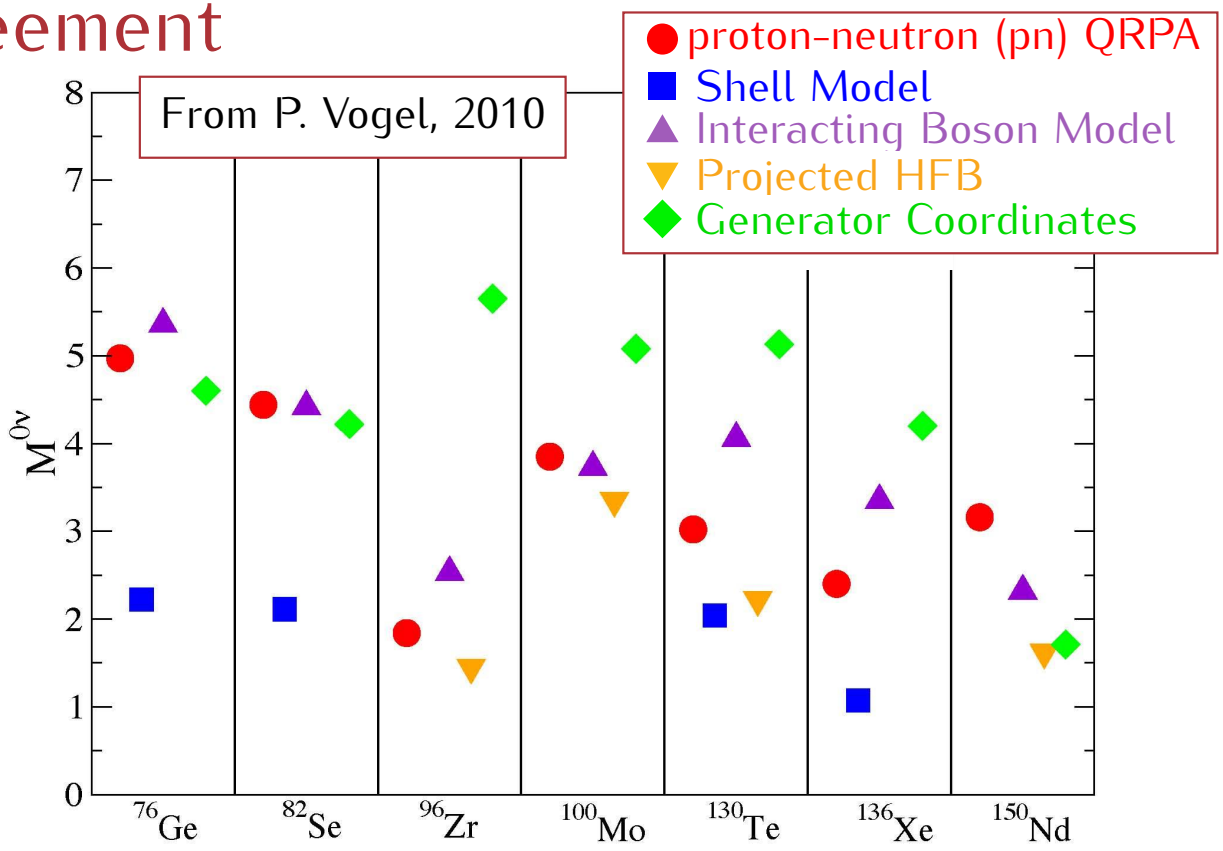
$$H(r, \overline{E}) \approx \frac{R}{r}$$

Corrections are from “forbidden” terms, weak nucleon form factors, two-body currents…

Recent Level of Agreement

Same level of agreement in 2014.

Not so great.



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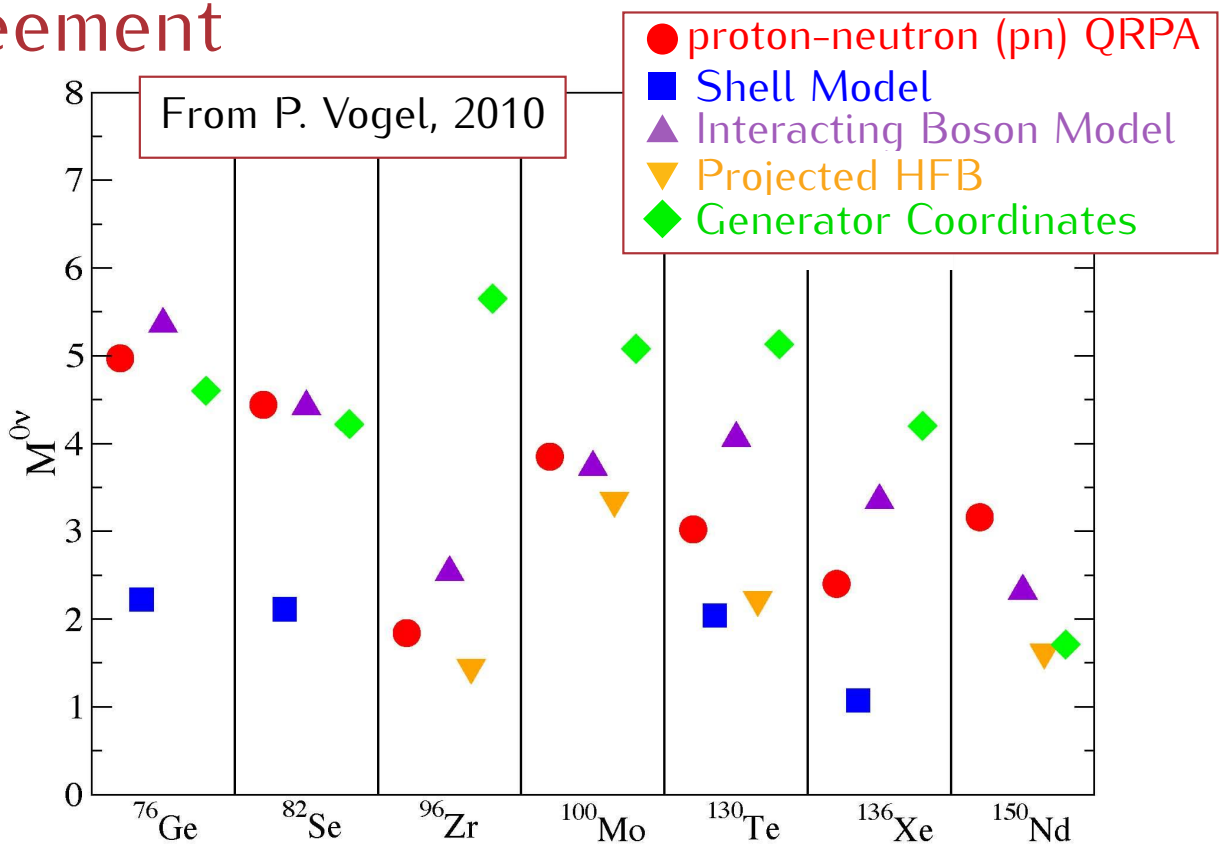

Main Message

More computing power and new many-body methods have recently generated major progress in nuclear structure theory.

We have the opportunity to improve all the models above, connect them to *ab initio* work. Should push them all as far as possible.

Will require funding, leadership class computing.

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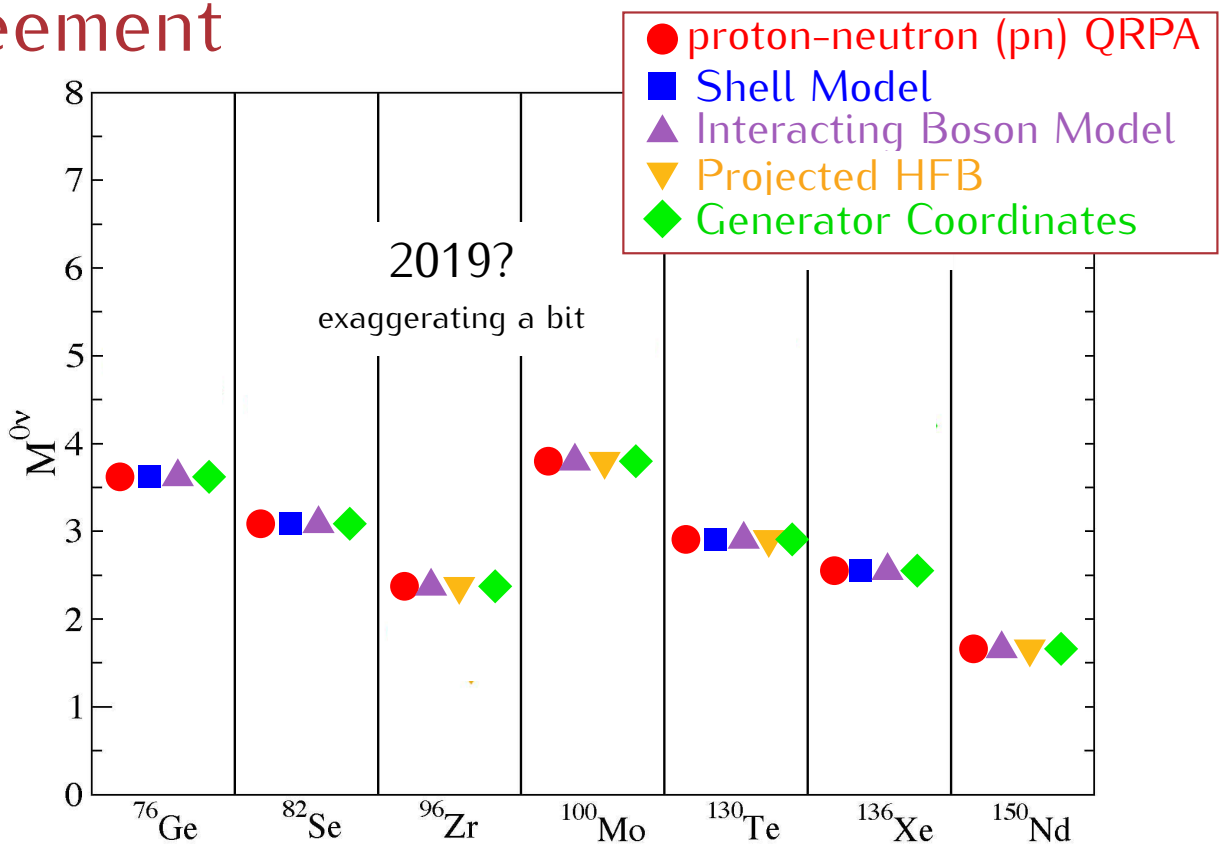

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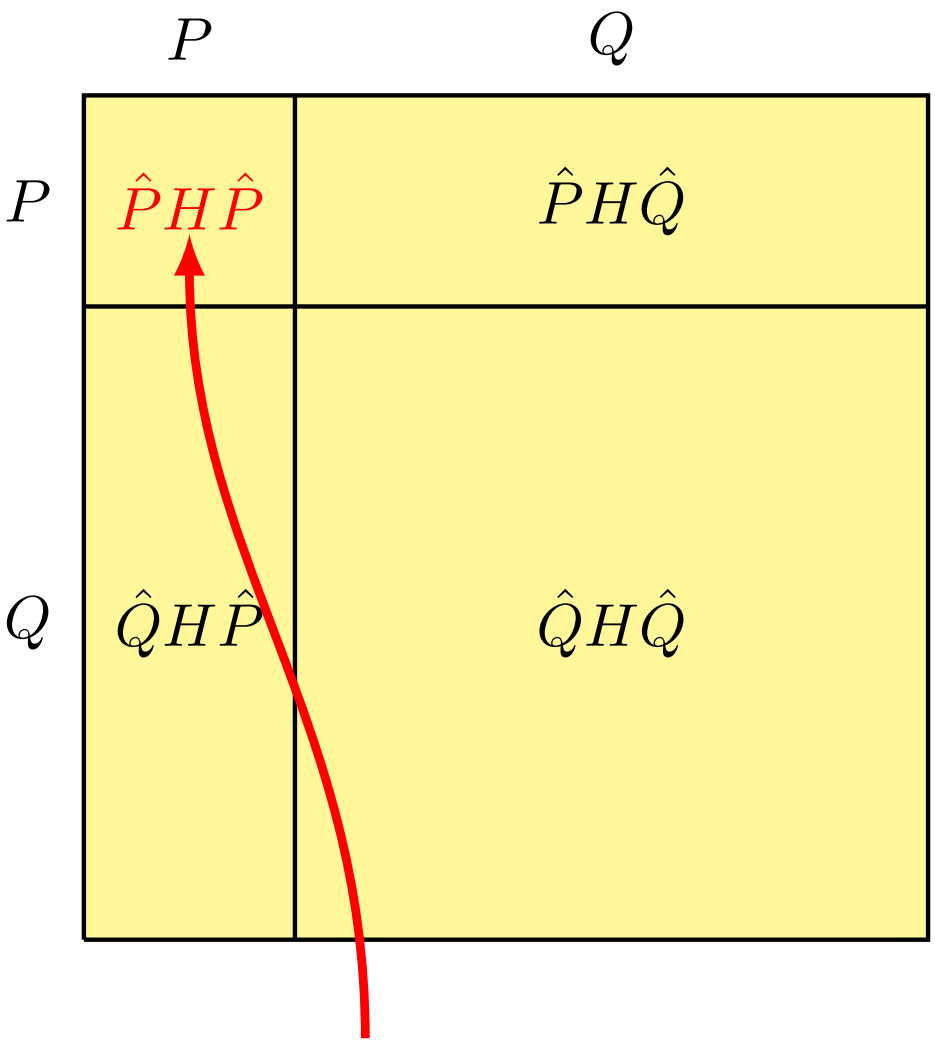
"Ab Initio" Shell Model

Partition of Full Hilbert Space



Shell model done here

P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing d most important eigenvalues.

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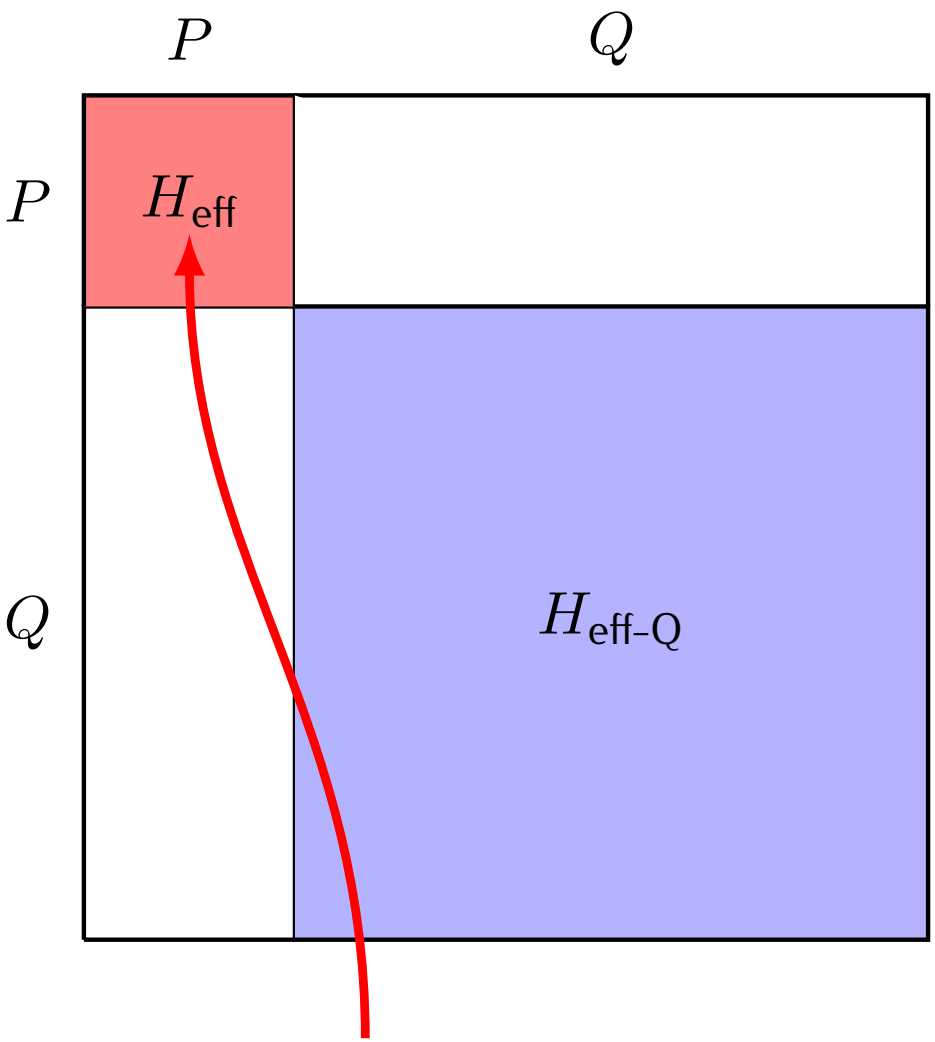

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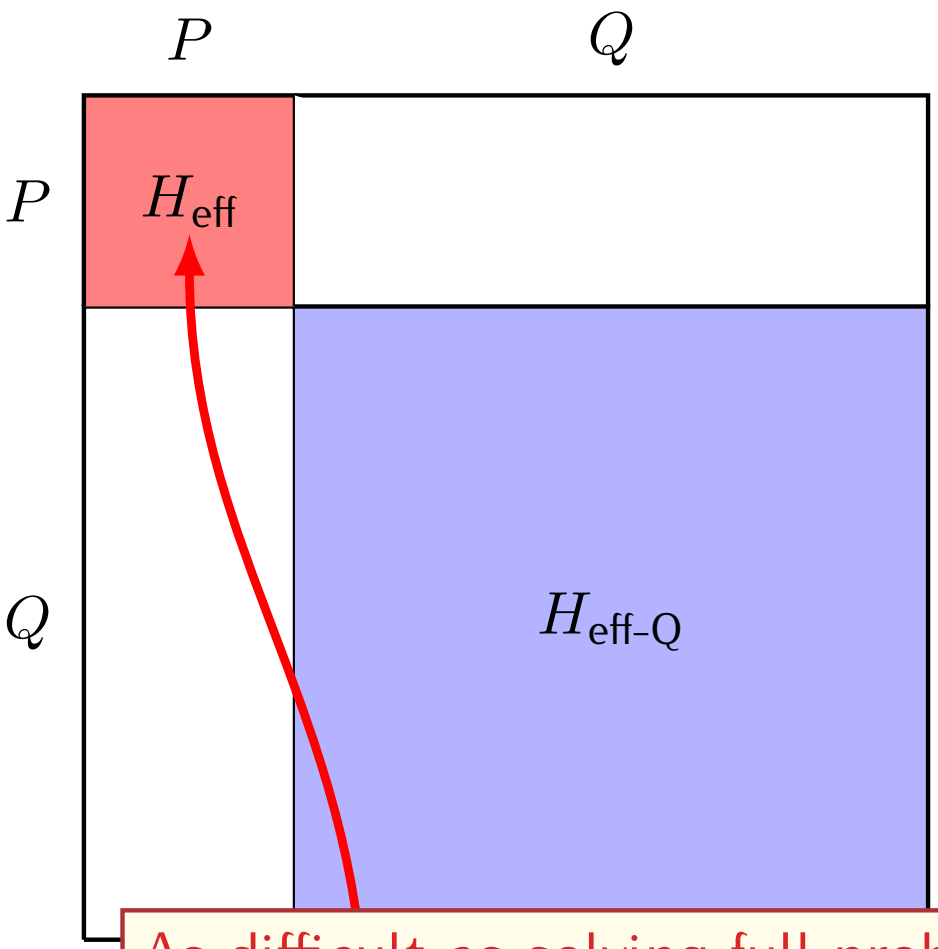

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For transition operator $\hat{M}$, must apply same transformation to get $\hat{M}_{\text{eff}}$.

As difficult as solving full problem. But idea is that N-body effective operators may not be important for N > 2 or 3.

Shell model done here

Some Options

- **Coupled Clusters:**
  1. Calculate low-lying spectra of ^{56}Ni + 1 and 2 nucleons (and + 3 nucleons in some approximation).
  2. Do Lee-Suzuki mapping of lowest eigenstates onto $f_{5/2}pg_{9/2}$ shell, determine effective Hamiltonian and decay operator.
  3. Use these operators in shell-model calculation of matrix element for ^{76}Ge (with analogous plans for other elements).

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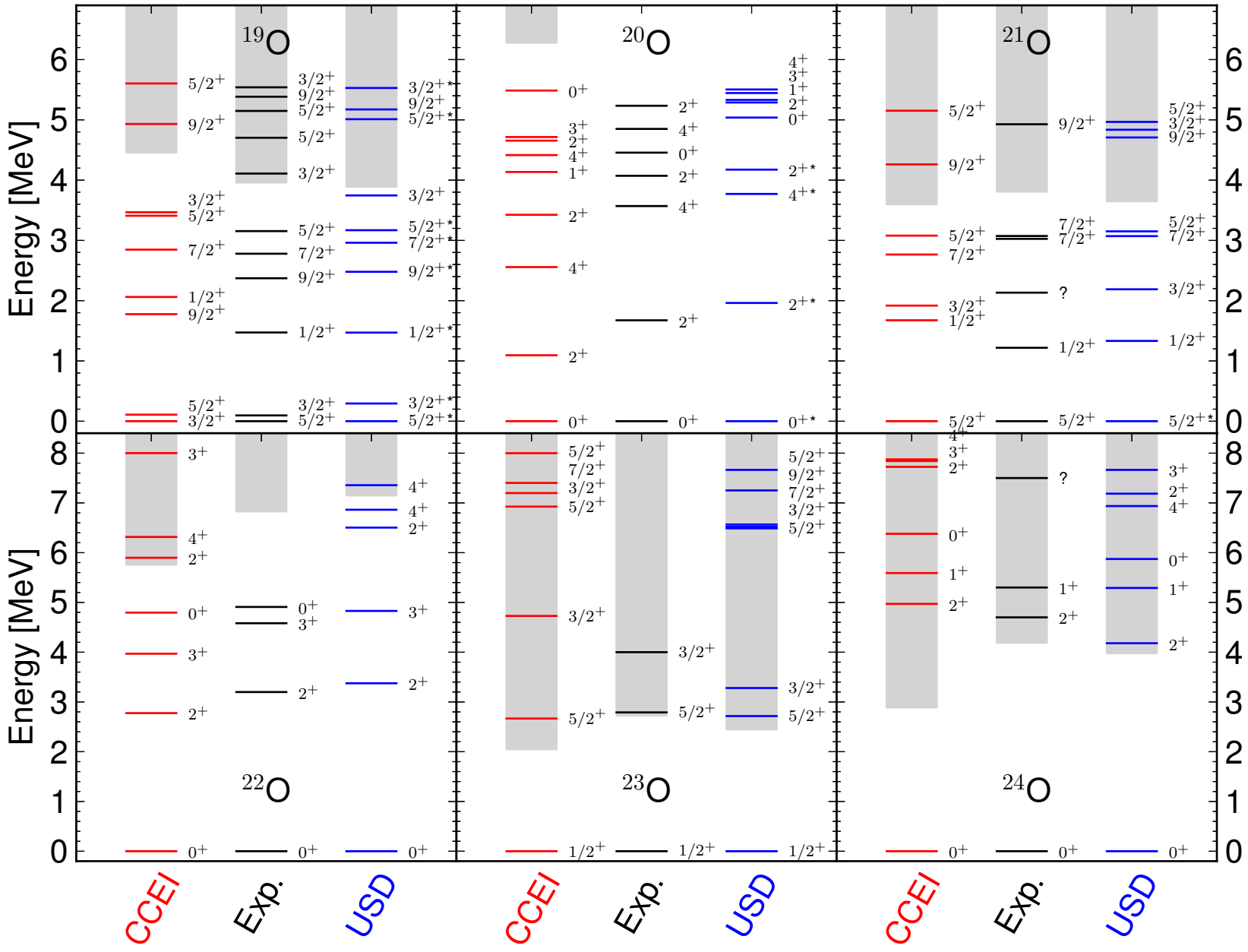
- **No-Core Shell Model and Monte-Carlo Schemes:** Possible in principle; A=76 too large?

Coupled Clusters in sd Shell: Oxygen



Improving QRPA: Make More Fermionic

Problem: Calculation of overlap between states in different nuclei not possible within strict QRPA, which only provides transition density. Need real expressions for states.

Terasaki and Delion/Schuck both extend QRPA boson vacuum to quasiparticle coupled-cluster like fermionic state:

$$|\text{“}QRPA\text{”}\rangle \propto \exp\left(Z_{pp'nn'}\alpha_p^\dagger\alpha_{p'}^\dagger\alpha_n^\dagger\alpha_{n'}^\dagger\right)|HFB\rangle \ .$$

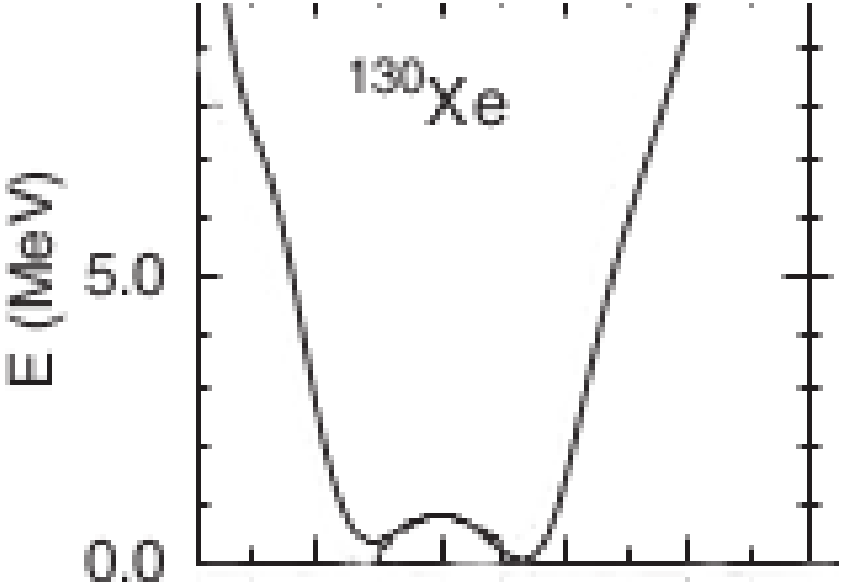
Terasaki finds correction to norm that alters $\beta\beta$ matrix element in large-scale calculation. Delion and Schuck derive self-consistent QRPA with renormalized phonons (not yet applied to $\beta\beta$ decay).

Ideas need to be explored more carefully, made consistent with DFT, which also deemphasizes explicit states.

GCM/IBM: Include All Important Degrees of Freedom

Generator Coordinate Method is perhaps best approach if nuclei don't have definite shape, can't be approximated by single mean field.

Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle$. Then diagonalize H in space of symmetry-restored quasiparticle vacua with different $\langle Q_0 \rangle$.

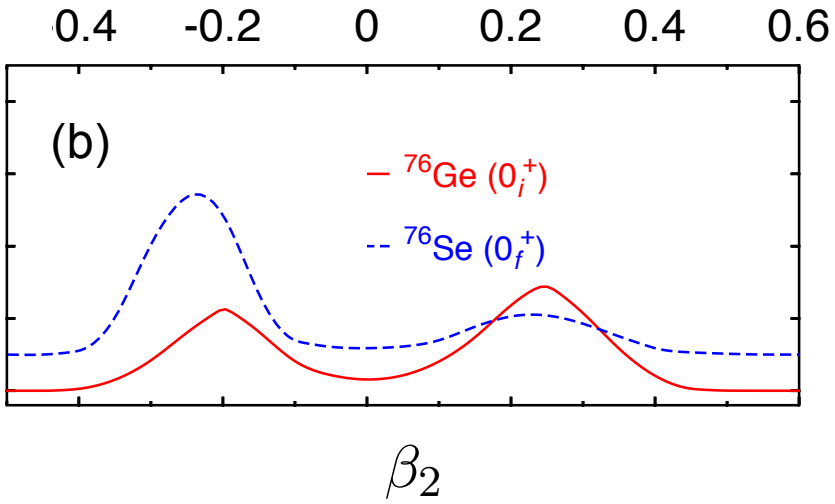


β_2 = deformation

Robledo et al.:

Energy minima at $\beta_2 \approx \pm .15$

Collective wave functions



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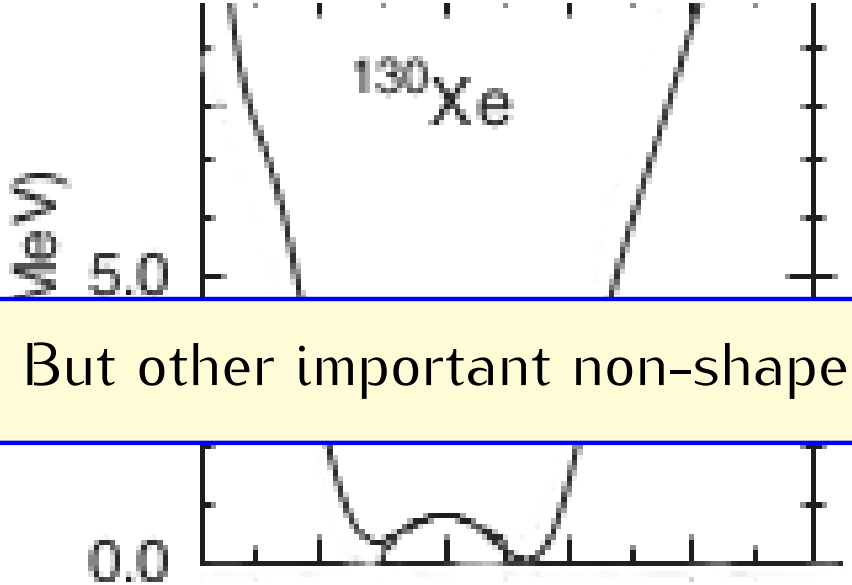

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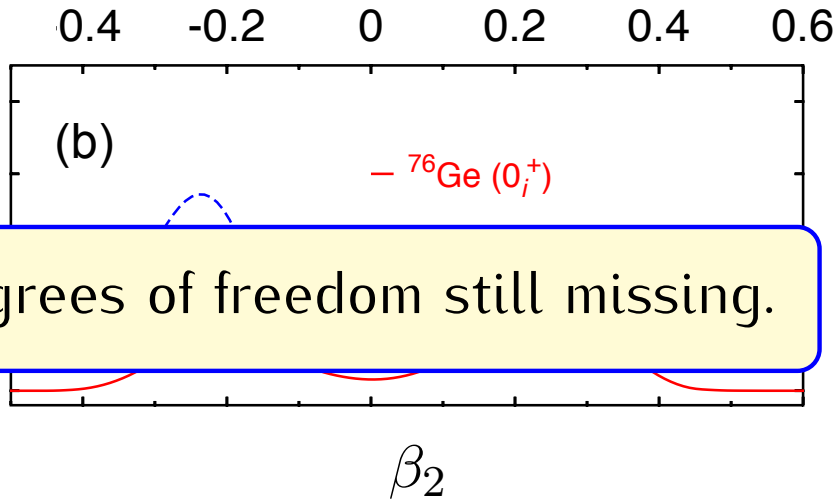

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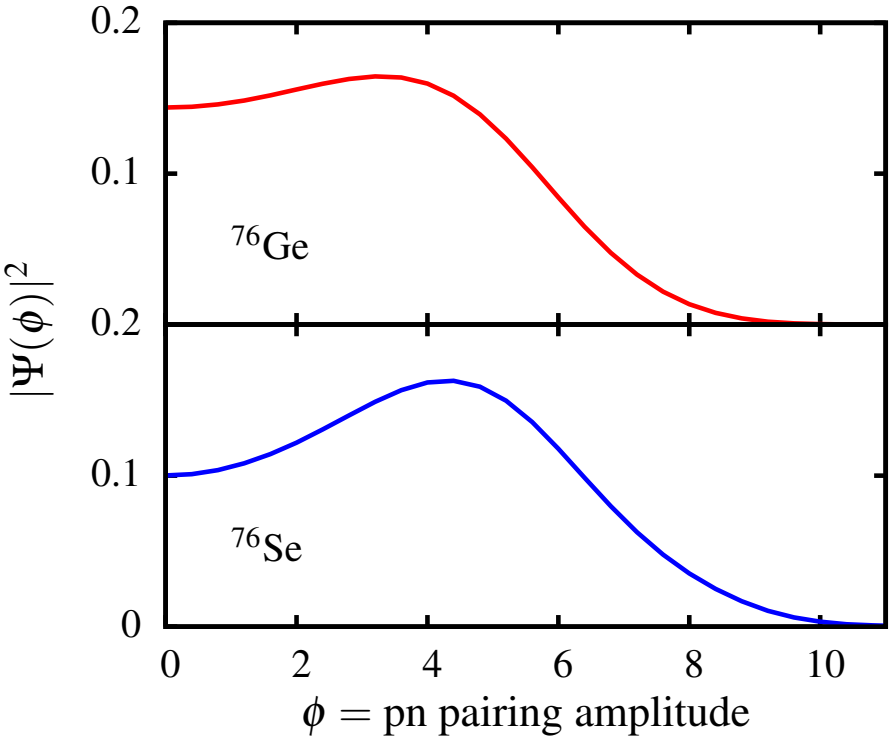
But other important non-shape degrees of freedom still missing.

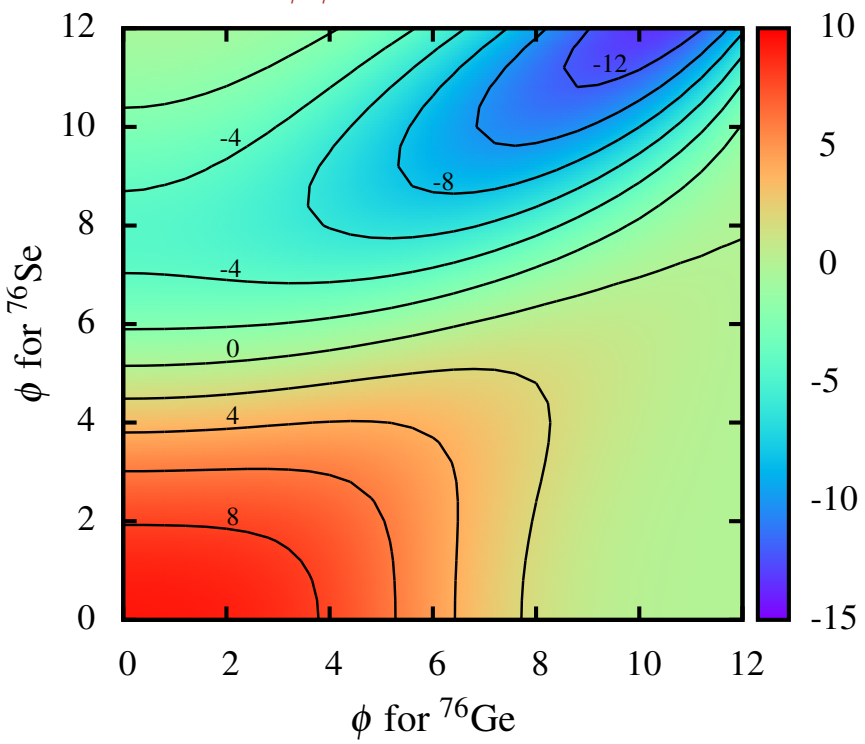
GCM Example: Proton-Neutron (pn) Pairing

Can build pn correlations into mean field. Frozen out in mean field minimum, but included dynamically in GCM.

Collective pn-pairing wave functions



$0\nu\beta\beta$ matrix element



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Proton-neutron pairing significantly reduces matrix element.

Next steps: combine with DFT or *ab initio* interaction, include non-collective degrees of freedom (jacking up computing needs).

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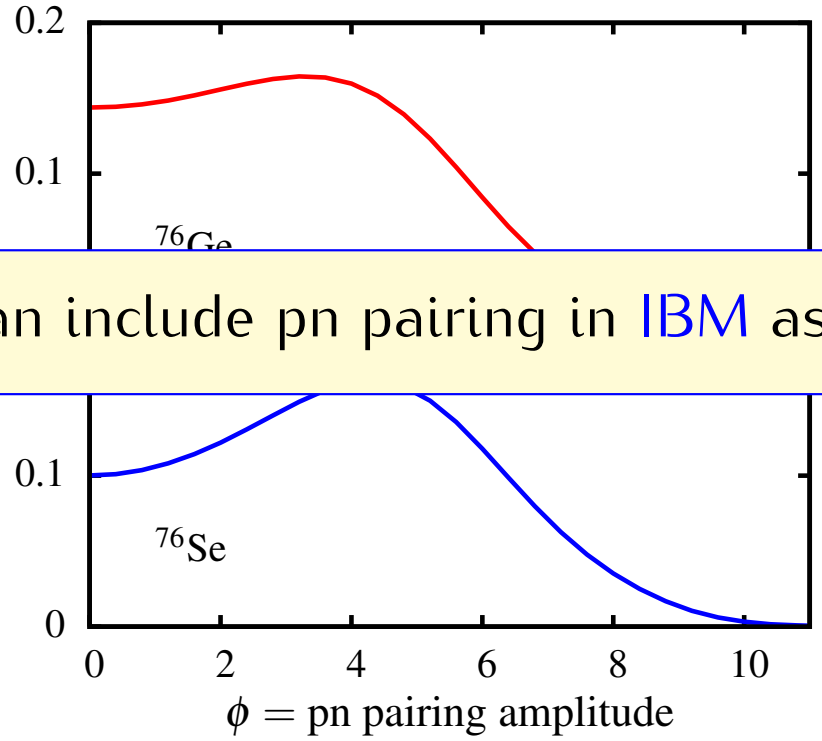

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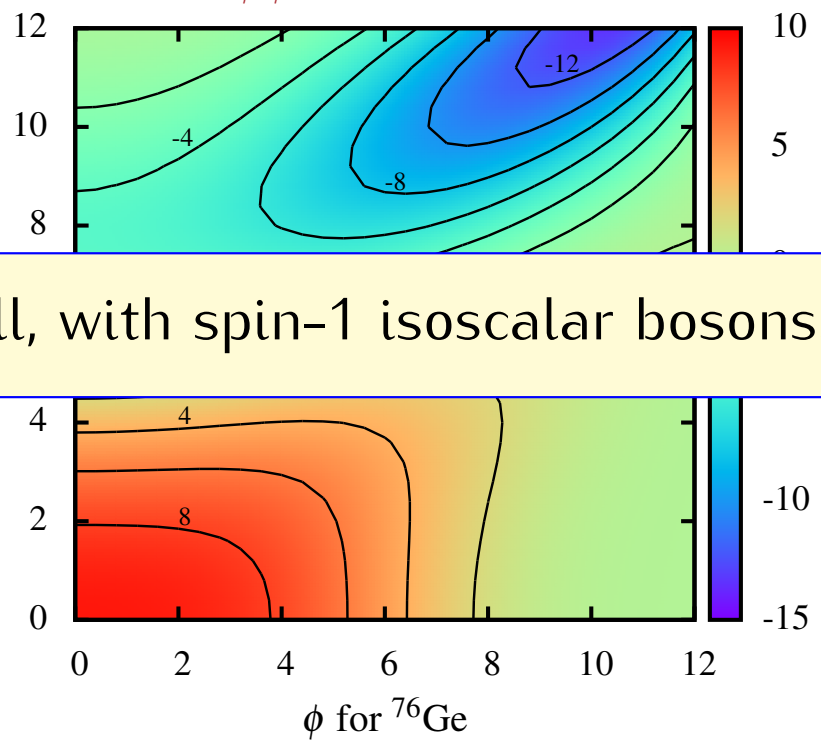

Can include pn pairing in IBM as well, with spin-1 isoscalar bosons.

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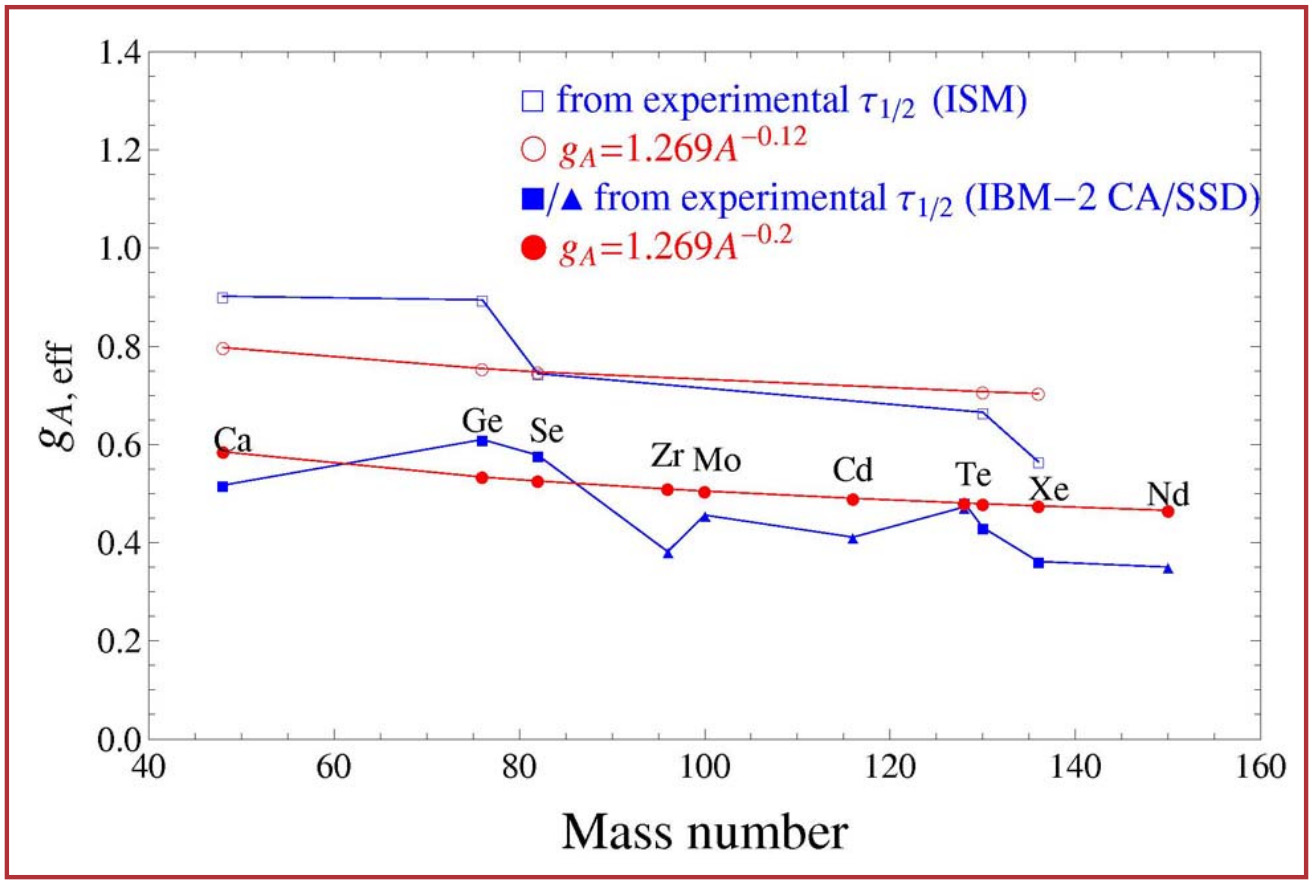
Issue Facing All Models: "g_A"

40-year-old problem particularly important in $\beta\beta$ decay

Effective g_A needed for two-neutrino decay in shell model and IBM-2



F. Iachello, MEDEX'13 meeting

If neutrinoless matrix elements quenched by same amount, experiments are in trouble; rates go like $(g_A)^4$.

Resolving the Issue

Typical practice: "Renormalize" g_A only for 2ν decay. Assume 0ν decay unaffected.

Better practice: Understand reasons for over-prediction of β and 2ν $\beta\beta$ rates. Must be due to

1. Many-body weak currents, either modeled explicitly as π, ρ exchange, etc., or from effective-field-theory fits. Conventional wisdom says meson-exchange effects in β decay are small; chiral-EFT folk suggest they may not be.

   More careful EFT work, in progress, should settle question.

2. Truncation of model space.

   Will be fixed, e.g., in ab-initio shell model.

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   Ths will all be straightened out soon.

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Confronting Data

All methods will allow calculation of many other observables. Community should decide on a standard set, periodically assess published calculations and inform experimentalists.

SciDAC NUCLEI collaboration is attacking $\beta\beta$ decay from several angles; will assess state of art at annual meetings.

Finally...

Theory has developed tools that promise, sincerely, to reduce the uncertainty in the matrix elements.

Recent progress in *ab initio* work and density-functional theory will inform and constrain future calculations.

Investment of targeted resources — into person-power, collaborations, computing power[1] — will speed the process significantly.

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That's all; thanks.

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